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MODULAR CENTERS OF ADDITIVE LATTICES

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The modular center of a lattice L $(\mathfrak{M}(L))$ is defined to be $\{x \mid x \in L$ and $\mathfrak{M}(a,x)$ for all $a \in L\}$ where the symbol $\mathfrak{M}(a,x)$ means that a and x form a modular pair. A lattice L is said to be <u>additive</u> iff whenever p is an atom of L such that $p \leq x \vee y$, then there exists atoms x_1 and y_1 in L with $x_1 \leq x$ and $y_1 \leq y$ such that $p \leq x_1 \vee y_1$. The lattice L of convex subsets of a vector space V over an ordered division ring is additive, and in this case $\mathfrak{M}(L)$ is the affine subsets of V.

If L is atomistic and additive, then $\mathfrak{M}(L)$ is a complete lattice in its own right, with the meet operation in $\mathfrak{M}(L)$ being the meet operation in L.

We present conditions in L which guarantee that $\mathfrak{M}(L)(p,1)$ is a projective geometry whenever p is an atom of L, and then give conditions on L which imply that $\mathfrak{M}(L)$ is the affine subsets of a vector space V over a decision ring R. In the latter case we show further that the R is ordered and that L is the lattice of the convex subsets of V.

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